The Fujita type a critical exponent for a double nonlinear parabolic equation and system

Mersaid ARIPOV [1]

[1] *Department of Informatics and Computer Analysis, National University of Uzbekistan, Tashkent, Uzbekistan*
E-mail: mirsaidaripov@mail.ru

Abstract: Consider in $Q = \{(\mathrm{t}, \mathrm{x}) : \mathrm{t} > 0,\ x \in R^N\}$the following problem Cauchy to a degenerate parabolic system with double nonlinearity

$$(1) \qquad \begin{cases} \frac{\partial u}{\partial t} = \nabla \left(v^{m_1 - 1} \left| \nabla u \right|^{p-2} \nabla u \right) + div(c(t)u) + u^{\beta_1}, \\ \frac{\partial v}{\partial t} = \nabla \left(u^{m_2 - 1} \left| \nabla v \right|^{p-2} \nabla v \right) + div(c(t)v) + v^{\beta_2} \end{cases}$$

$$(2) \qquad u(0, x) = u_0(x) \geq 0,\ v(0, x) = v_0(x) \geq 0,\ x \in R^N$$

This problem describes the different nonlinear processes [1-4]. The solution of the problem (1), (2) depending on value of numerical parameters m_i, β_i , $i = 1, 2, p$and functions $c(t)$, $\gamma(t)$ have a property of a finite speed perturbation, space localization, blow-up and so on [1,2]. Now these properties of the solution to the problem Cauchy for a different nonlinear equation in the case single equation and system intensively studied by many authors (see [1-4] and the literature therein). Many works (see [1-3] and literature therein) are devoted to the Fujita type critical exponents for semi-linear system (1) ($m_i = 1,\ \beta_i$, $i = 1, 2, p = 2$). In this case in [4] the following condition $(\beta_i + 1)/\beta_1\beta_2 - 1 = N/2,\ i = 1, 2$ of the critical exponents were established.

In this work an algorithm for establishing value of a critical exponent for the system (1) and for some other degenerate system based on approximately self similar approach is suggested. Using comparison principle the condition of a global solvability, property a finite speed of perturbation, space localization of the solution, blow-up is established. The following condition of a global solvability
$\tau(t)\bar{u}^{\beta_1 - (p-2)}\bar{v}^{-(m_1 - 1)} < (\beta_1 - 1)N/p,\ \ \tau(t)\bar{u}^{-(m_2 - 1)}\bar{v}^{\beta_2 - (p-2)} < (\beta_2 - 1)N/p,\ where$
$\bar{u}(t) = (T + \int \gamma(t)dt)^{-1/(\beta_1 - 1)},\ \bar{v}(t) = (T + \int \gamma(t)dt)^{-1/(\beta_2 - 1)},\ T \geq 0, \tau(t) = \int [\bar{u}(t)]^{(p-2)} [\bar{v}(t)]^{(m_1 - 1)} dt$
which consists in particular the result of the work [4] is established.

Keywords: double nonlinear, parabolic system, property, critical exponent

2010 Mathematics Subject Classification: 35B20, 35B33

References

[1] Samarskii A.A., Galaktionov V.A., Kurdyomov S.P., Mikhailov A.P., “Blow-up in quasilinear parabolic equations", *Berlin, Walter de Grueter*, 4, 535 p., 1995.

[2] Aripov M., Sadullaeva Sh.A., “An asymptotic analysis of a self-similar solution for the double nonlinear reaction-diffusion system", *J. Nanosystems: physics, chemistry, mathematics*, 6(6), pp. 793-802, 2015.

[3] Aripov M., Sadullaeva Sh.A., “Qualitative properties of solutions of a doubly nonlinear reaction-diffusion system with a source", *Journal of Applied Mathematics and Physics*, 3, pp. 1090-1099, 2015.

[4] ESCOBEDO M.AND. HERRERO M. A, “Boundedness and Blow Up for a Semilinear Reaction-Diffusion System", *JOURNAL OF DIFFERENTIAL EQUATIONS*, 89, pp. 176-202, 1991.