# On parabolic inverse problems with overdetermination data on spatial manifolds

Sergey Pyatkov

*Yugra State University, Khanty-Mansiisk, Russia*
*E-mail: pyatkov@math.nsc.ru*

**Abstract:** We examine the question on finding a solution to a parabolic system together with a right-hand side and coefficients in parabolic systems of equations. Let $G$ be a domain in $\mathbb{R}^n$ with a boundary $\Gamma$ of class $C^{2m}$ and $Q = (0,T) \times G$. The parabolic system is of the form

$$u_t + A(t,x,D)u = g = \sum_{i=1}^{r} b_i(t,x) q_i(t,x') + f, \quad (t,x) \in Q, \tag{1}$$

where $x' = (x_1, x_2, \dots, x_k)$, $x'' = (x_{k+1}, x_{k+2}, \dots, x_n)$ and $A$ is a matrix elliptic operator of order $2m$ with matrix coefficients of dimension $h \times h$, representable as

$$A(t,x,D) = \sum_{i=r+1}^{sh} q_i(t,x') A_i(t,x,D_x) + A_{hs+1}(t,x,D_x),$$
$$A_i = \sum_{|\alpha| \le 2m} a_{i\alpha}(t,x) D^{\alpha},$$
$$u|_{t=0} = u_0, \quad B_j u|_S = \sum_{|\beta| \le m_j} b_{j\beta}(t,x) D^{\beta} u|_S = g_j(t,x), \tag{2}$$

where $m_j < 2m$, $j = 1, 2, \dots, m$, and $S = (0,T) \times \Gamma$. The unknowns are a solution $u$ and functions $q_i(t,x')$ $(i = 1, 2, \dots, sh)$ occurring into the right-hand side of (1) and in the operator $A$. The overdetermination conditions are of the form

$$u|_{S_i} = \psi_i(t,x'), \ S_i = (0,T) \times \Gamma_i, \quad i = 1, \dots, s \tag{3}$$

where $\{\Gamma_i\}$ $(i = 1, 2, \dots, s)$ is a set of smooth $n-1$-dimensional surfaces lying in $G$. Under certain conditions on the data, we establish the existence and uniqueness theorems for solutions to these inverse problems.

**Keywords:** inverse problem, parabolic system, existence, uniqueness

**2010 Mathematics Subject Classification:** 35R30, 35K41, 35K35