The resolvent equation of nonlinear Fredholm integral equations

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Abstract: It is considered the new method solving of nonlinear Fredholm integral equation. It is found the sufficient condition for the existence of a solution of the nonlinear integral equation and it is proved uniqueness of the found solution.

In the article we consider the nonlinear Fredholm integral equation [1, 2]

$$\varphi(x) = \lambda \int_a^b K\left(x, t, \varphi(t)\right)\, dt + f(x), \tag{1}$$

where $f(x)$ is known continuous function defined on the $[a, b]$; Kernel $K\left(x, t, \varphi(t)\right)$ nonlinearly depends on the unknown function $\varphi(x)$ and continuous on set of arguments $x \in [a, b]$, $t \in [a, b]$,$\varphi \in (c, h)$, and it has a continuous derivative with respect to functional variable φ; λ is a parameter.

By the method described in [3, 4] we have established that if the kernel $K\left(x, t, \varphi(t)\right)$ satisfies

$$0 \equiv \int_a^b \int_a^b K'_\varphi\left[x, t, \psi(t, \lambda)\right] u(t)\, dt\, dx, \tag{2}$$

the equation (1) admits a solution of the form

$$\varphi(x) = \varphi_0(x) + \lambda u(x), \tag{3}$$

where $\varphi_0(x)$, $u(x)$, $\psi(t, \lambda)$ are known functions . This method is the most effective in the case where identity (2) can be represented in the form

$$0 \equiv A(x) \int_a^b G\left[t, \psi(t, \lambda)\right] \bar{u}(t)\, dt. \tag{4}$$

In this case, if the parameter λ is *the resolvent number*, i.e. is the root of *the resolvent algebraic equation*

$$B(\lambda) = \int_a^b G\left[t, \psi(t, \lambda)\right] \bar{u}(t)\, dt = 0, \tag{5}$$

correlation (2) holds for any $x \in [a, b]$.

Substituting the found value for resolvent number $\lambda_1, ..., \lambda_p,\ p \leq \infty$ into (3) we obtain the solution of equation (1). We note that if λ is not a characteristic number of kernels $K'_\varphi\left[x, t, \psi(t, \lambda)\right]$, the solution is unique.

Keywords: nonlinear integral equation, uniqueness of solution, characteristic number, orthogonality

2010 Mathematics Subject Classification: 45B05

References

[1] Smirnov, N.S. *Vvedenie v teoriyu nelineynih integralnih uravneniy.* M.: ONTI, 1936. (in Russian)

[2] Krasnov, M. L. *Integralnie uravneniya.* M.: Nauka, 1975. (in Russian)

[3] Kerimbekov, A. "About the one method of solving nonlinear Fredgolm integral equations", *Abstract. V Congress of the Turkic World mathematicians, Kyrgyzstan, "Issyk-Kul Aurora", 5-7 june* , P.84, 2014.

[4] Kerimbekov, A. "Metod resheniya nelineynih integralnih uravneniy Fredgolma", *Vestnik KRSU* , Vol.16, No.1, pp. 23-26, 2016. (in Russian)