Noncommutative Holder-type inequalities

Turdebek N. Bekjan[1]

[1] Department of Fundamental Mathematics, L. N. Gumilyov Eurasian National University, Ȧstana, Kazakhstan
E-mail: bekjant@yahoo.com

Abstract: We generalize some known Holder-type inequalities of matrix to the measurable operators associated with semi-finite von Neumann algebra $\mathcal{M}$ case.

In [4], Horn and Zhan have shown that if p, q and r are positive real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$, then

$$(1) \qquad \||\,|A^*B|^r\,|\| \leq \||\,|A|^{pr}\,|\|^{\frac{1}{p}} \||\,|B|^{qr}\,|\|^{\frac{1}{q}}$$

for $A, B \in B(H)$ and for unitarily invariant norms. Hiai and Zhan [2] proved that if A, B, C and D in $B(H)$, then

$$(2) \qquad 2^{|\frac{1}{p}-\frac{1}{2}|}\||\,|C^*B + D^*B|\,|\| \leq \||\,|A|^p + |B|^p\,|\|^{\frac{1}{p}}\||\,|C|^q + |D|^q\,|\|^{\frac{1}{q}}$$

for all positive real numbers p and q such that $\frac{1}{p} + \frac{1}{q} = 1$.

In this talk, we will use the concept of uniform Hardy-Littlewood majorization studied by Kalton and Sukochev [5], and their main result, to generalize the Holder type inequalities (1) and (2) for τ-measurable operators associated with semi-finite von Neumann algebra $\mathcal{M}$ and for symmetric Banach spaces norm.

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