Solution of a Cauchy problem for iterated generalized two-axially symmetric equation of hyperbolic type

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Abstract: In the work an explicit solution of Cauchy problem for iterated generalized two-axially symmetric equations of hyperbolic type was constructed.

In this work, we considered an analogue of the Cauchy problem in a domain $\Omega = \{(x, y) : \ 0 < y < x\}$ finding a classical solution $u(x, y) \in C^{2m-1}(\bar{\Omega})$ of the equation

$$[h^{\lambda}_{\beta,\alpha}]^m(u) \equiv \left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2} - \frac{2\alpha}{x}\frac{\partial}{\partial x} + \frac{2\beta}{y}\frac{\partial}{\partial y} + \lambda^2\right)^m u(x, y) = 0, \qquad (1)$$

satisfying the initial conditions

$$\left.\frac{\partial^{2k} u}{\partial y^{2k}}\right|_{y=0} = \varphi_k(x), \ x \geqslant 0; \quad \left.\frac{\partial^{2k+1} u}{\partial y^{2k+1}}\right|_{y=0} = 0, \ x \geqslant 0, \ \ k = \overline{0, m-1}; \qquad (2)$$

where $\alpha, \beta, \lambda \in R$, $m \in N$, and $0 < \alpha < 1$, $0 < \beta < 1/2$, $\varphi_k(x)$, $(k = \overline{0, m-1})$ are given smooth functions.

By applying two-dimensional generalized Erdelyi-Kober operator [1], the explicit formula of a solution of the problem $\{(1), (2)\}$ was received

$$u(x, y) = \sum_{n=0}^{m-1} \frac{2^{-2n} y^{1-2\beta}}{n!\Gamma(\beta + n)} \int\limits_{x-y}^{x+y} f_n(s) \left(\frac{s}{x}\right)^{\alpha} \frac{\Xi_2(\alpha, 1-\alpha; \beta + n; \sigma, \omega)}{[y^2 - (x-s)^2]^{1-\beta-n}} ds,$$

where $f_n(x) = \sum\limits_{j=0}^{n} (-1)^j C_n^j [B^x_{\alpha-1/2}]^j \tilde{\varphi}_{n-j}(x)$, $\tilde{\varphi}_n(x) = \sum\limits_{j=0}^{n} \gamma_n C_n^j \lambda^{2(n-j)} \varphi_j(x)$, $\gamma_n = \Gamma((2n+1)/2 + \beta)/\Gamma((2n+1)/2)$, $\sigma = [y^2 - (s-x)^2]/(4xs)$, $\omega = -(\lambda^2/4)[y^2 - (s-x)^2]$, $\Xi_2(\alpha, 1-\alpha; \beta+n; \sigma, \omega)$ is Humbert hypergeometric function.

Keywords: Erdélyi-Kober operator, two-axially symmetric equations

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References

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