Necessary and sufficient condition that function belong to the Nikolskii-Morrey space $H^{(r)}M^{\lambda}_{p\,x_1}$.

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Abstract: In this paper there were established the necessary and sufficient conditions that the function belongs to the Nikolskii-Morrey space $H^{(r)}M^{\lambda}_{p\,x_1}$.

Definition 0.1. *Let $0 < p \le +\infty$, $0 \le \lambda \le \frac{n}{p}$. Denote by $M_p^{\lambda}(\mathbb{R}^n)$ the space of all real-valued function f, measurable on $\mathbb{R}^n$, such that*

$$\|f\|_{M_p^{\lambda}(\mathbb{R}^n)} = \sup_{\substack{x\in\mathbb{R}^n \\ \rho>0}} \|\rho^{-\lambda}\|f\|_{L_p(B(x,\rho))}\|_{L_\infty(0,\infty)} < \infty.$$

Definition 0.2. *We say measurable function $f(x_1,\dots,x_n)$ belongs to the class $H^{(r)}M^{\lambda}_{p\,x_1}(B)$ $(r>0,\ 1\le p\le +\infty)$, if it satisfies the following condition.*

Assume that $r=\rho+\alpha$, where ρ is integer and $0<\alpha\le 1$. Let for almost all $x_2,\dots,x_n$ exists the partial derivative $\frac{\partial^\rho f}{\partial x_1^\rho}$ defined almost everywhere on $\mathbb{R}^n$ that belong to the Morrey space $M_p^{\lambda}(\mathbb{R}^n)$ and such that for all h

$$\left\|\frac{\partial^\rho f(x_1+h,x_2,\dots,x_n)}{\partial x_1^\rho} + 2\frac{\partial^\rho f(x_1,\dots,x_n)}{\partial x_1^\rho} + \frac{\partial^\rho f(x_1-h,x_2,\dots,x_n)}{\partial x_1^\rho}\right\|_{M_p^{\lambda}(\mathbb{R}^n)} \le B|h|^\alpha.$$

Definition 0.3. *Denote by $A_{\nu\,x_1}(f;M_p^{\lambda}(\mathbb{R}^n))$ the best approximation of function $f\in M_p^{\lambda}(\mathbb{R}^n)$ by entire functions g_ν of exponential type ν in metrics of the space $M_p^{\lambda}(\mathbb{R}^n)$, i.e.*

$$A_{\nu\,x_1}(f;M_p^{\lambda}(\mathbb{R}^n)) = \inf_{g_\nu}\|f-g_\nu\|_{M_p^{\lambda}(\mathbb{R}^n)}.$$

Theorem 0.4. *Function $f\in M_p^{\lambda}(\mathbb{R}^n)$ belongs to the class $H^{(r)}M^{\lambda}_{p\,x_1}$ (with some constant B), if and only if there exists the constant K such that*

$$A_{\nu,x_1}(f;M_p^{\lambda}(\mathbb{R}^n)) < \frac{K}{\nu^r},$$

for all $\nu\ge 1$ or for all ν that run over geometric increasing progression $\nu=a^k$ $(a>1,\ \ k=0,1,\dots)$.

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