Existence and nonexistence of positive solutions for singular nth-order three-point nonhomogeneous boundary value problem

Mostepha NACERI [1], Amir ELHAFFAF [2]

[1] *Economics,Commercial and Management Sciences,Preparatory School of Oran-Algeria*
E-mail: nacerimostepha@yahoo.fr
[2] *Mathematics Faculty Of Science,Oran University,BP1524,Es-Senia Algeria*
E-mail: elhaffaf1@yahoo.com

Abstract: In this article, we consider the boundary value problem $u^{(n)}(t) + f(t, u(t)) = 0$, $0 < t < 1$, subject to the boundary conditions $u(0) = u'(0) = 0, ..., u^{(n-3)}(0) = u^{(n-2)}(0) = 0$ and $u^{(n-2)}(1) - \alpha u^{(n-2)}(\eta) = \lambda$. In the setting, $0 < \eta < 1$ and $\alpha \in [0, \frac{1}{\eta})$ are constants and $\lambda \in [0, +\infty)$ is parameter. By placing certain restrictions on the nonlinear term f, we prof the existence and nonexistence of at least one positive solution to the boundary value problem with the use of the Krasnosel'skii fixed point theorem. The novelty in our setting lies in the fact that $f(t, u)$ may be singular at $t = 0$ and $t = 1$. We conclude with examples illustrating our results obtained in this paper.

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