The positivity of the difference operator with periodic conditions and its applications

Allaberen ASHYRALYEV [1], Fatih Sabahattin TETİKOGLU [2]

Emeritus Professor, Turkey & Turkmenistan

E-mail: aallaberen@gmail.com, fstetik@gmail.com

Abstract: In this study, the second order of approximation of the difference operator A_h^x approximates the second order differential dependent operator A^x defined by the formula

$$A^x u = -a(x)u_{xx}(x) + \delta u(x), \delta \geq 0, a(x) = a(x+2\pi), x \in \mathbb{R}^1$$

with domain

$$D(A^x) = \left\{ u(x) : u(x), u'(x), u''(x), u(x) = u(x+2\pi), x \in \mathbb{R}^1, \int\limits_0^{2\pi} u(x)dx = 0 \right\}$$

is presented. The Green's function of the difference operator A_h^x is constructed. The estimates for the Green's function are obtained. The positivity of operator A_h^x in the Banach space $C(\mathbb{R}_{1h})$ is established. It is proved that for any $\alpha \in (0, \frac{1}{2})$, the norms in spaces $E_\alpha = E_\alpha(C(\mathbb{R}_{1h}), A_h^x)$ and $C^{2\alpha}(\mathbb{R}_{1h})$ are equivalent uniformly with respect to h. The positivity of the operator A_h^x in Hölder spaces of $\mathring{C}^{2\alpha}(\mathbb{R}_{1h})$, $\alpha \in (0, \frac{1}{2})$ is proved. In applications, theorems on well-posedness for difference schemes of the approximate solution of local and nonlocal boundary value problems for elliptic differential equations are presented.

Keywords: positivity of difference operators, periodic boundary conditions, Green's function.

2010 Mathematics Subject Classification: 34K13, 34B09,

REFERENCES

[1] Sobolevskii P.E., "A New Method of Summation of Fourier Series Converging in C-Norm", *Semigroup Forum*, vol 71, p 289-300 (2005).

[2] Ashyralyev A. , Tetikoglu F. S., "The positivity of the second order difference operator with periodic conditions in Hölder spaces and its applications" *International Conference on Analysis and Applied Mathematics (ICAAM 2014), AIP Conf. Proc,* 1611, 336-343 (2014); doi: 10.1063/1.4893857.

[3] Ashyralyev A. , Tetikoglu F. S., "A note on fractional spaces generated by the positive operator with periodic conditions and applications" *Boundary Value Problems,* vol 2015(31), p 1-17, (2015); doi:10.1186/s13661-015-0293-9.