# Cauchy problem for a nonlinear degenerate parabolic system not in divergence form

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**Abstract:** In this paper, we consider the following Cauchy problem to a nonlinear degenerate parabolic system not in divergence form:

$$\begin{array}{l} \frac{\partial u}{\partial t} = v^{m_1} \nabla \left( \left| \nabla u^k \right|^{p-2} \nabla u^n \right), \\ \frac{\partial v}{\partial t} = u^{m_2} \nabla \left( \left| \nabla v^k \right|^{p-2} \nabla v^n \right), \end{array} \quad t > 0, \ x \in R^N,$$

$$u|_{t=0} = u_0(x) \geq 0, \quad v|_{t=0} = v_0(x) \geq 0, \ \ \forall x \in R^N$$

where $p, \ m_1, \ m_2, \ k, \ n$ - are positive constants, $N \geq 1$.

This system can be used to describe the development of multiple groups in the dynamics of biological groups, where $u, \ v$ are the densities of the different groups (see [1]). In paper [1], Gao et al. considered parabolic systems not in divergence form in the case $n = 1$ and $p = 2$, with source and null Dirichlet boundary conditions. The local existence and uniqueness of classical solution are proved. Moreover, it is proved that all solutions exist globally with homogeneous Dirichlet boundary condition. Porous medium equations with local sources or with nonlocal sources subjected to nonlocal boundary conditions were studied (see [3,4]). They discussed the conditions of existence and blow-up.

In this paper, by using the comparison principle [5] and the standard equation method [2], we will obtain a global solvability of solutions of Cauchy problem and using the method of nonlinear splitting, we can construct an asymptotic representation of the solution as $t \to \infty$. We obtain the estimates of the solution of Cauchy problem. Results of computational experiments shows, that the self - similar solutions are very appropriate approximation and the iterative method basing on the method Picard is effective for the solution of nonlinear problems and leads to the nonlinear effects if we will use as initial approximation the solutions of self similar equations constructed by the method of nonlinear splitting and by the method of standard equation.

**Keywords:** nonlinear system, non-divergence form, global solutions, asymptotic behavior of solutions

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