Well-posedness of high order of accuracy difference scheme

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Abstract: The simply nonlocal boundary value problem was investigated for the first time by A.V. Bitsadze and A.A. Samarskii in the paper [1]. In recent years, [3]– [5], the Bitsadze–Samarskii type nonlocal boundary value problem and its generalizations for various differential and difference elliptic equations have been studied. In this work, the Bitsadze-Samarskii type nonlocal boundary-value problem with the integral condition

$$(1) \qquad \begin{cases} -\frac{d^2 u(t)}{dt^2} + Au(t) = f(t), \ 0 < t < 1, \\ u(0) = \varphi, \ u(1) = \int\limits_0^1 \rho(\lambda)u(\lambda)d\lambda + \psi \end{cases}$$

for the differential equation of elliptic type in a Hilbert space H with the self-adjoint positive definite operator A with a closed domain $D(A) \subset H$ is considered. The well-posedness of the fourth order of accuracy difference scheme for approximate solution of nonlocal boundary-value problem (1) in Holder spaces with a weight is proved. The theoretical statements for the solution of this difference scheme are supported by the results of a numerical example.

Keywords: elliptic equation, nonlocal boundary value problem, well-posedness

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