On the stability of the telegraph equation with time delay

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Abstract: Telegraph equation is mostly interested in physical systems. Many physicists and engineers use telegraph equation without time delay [1]-[4]. However, for several reasons, in problems encountered in real life, time delay should be considered in modeling. In this study, the initial value problem for telegraph equations with delay

$$(1) \qquad \begin{cases} \frac{d^2v}{dt^2} + \alpha\frac{dv}{dt} + Av(t) = aAv([t]), t > 0, \\ \\ v(0) = \varphi, v'(0) = \psi \end{cases}$$

in a Hilbert space H with a self-adjoint positive definite operator A, is considered. Here $A \geq \delta I$, $\delta \geq \frac{\alpha^2}{4}$ and $0 \leq a < 1$. Theorem on stability estimates for the solution of this problem is established. As a test problem one-dimensional delay telegraph equation with Dirichlet boundary conditions is considered. Numerical solutions of this problem are obtained by first and second order of accuracy difference schemes.

Keywords: initial and boundary value problems, difference schemes, delay telegraph equations.

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