

# Determination of a control parameter of the $r$ -modified Crank-Nicholson difference scheme for the Schrödinger equation

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**Abstract:** The differential equations with parameters have been studied extensively by many researchers (see, [1]- [9] and the references therein). However, such problems were not well-investigated in general. In the present paper, the second order of accuracy  $r$ -modified Crank-Nicholson difference schemes are presented for the numerical solution of the boundary value problem for the Schrödinger differential equation with parameter  $p$

$$\begin{cases} i \frac{du(t)}{dt} + Au(t) + iu(t) = f(t) + p, & 0 < t < T, \\ u(0) = \varphi, & u(T) = \psi \end{cases}$$

in a Hilbert space  $H$  with self-adjoint positive definite operator  $A$ . The well-posedness of this difference schemes are established. The stability inequalities for the solution of difference schemes for three determination of a control parameter problems for the Schrödinger equation are obtained.

**Keywords:** determination of a control parameter problem, difference scheme, Schrödinger equation, stability, well-posedness

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