High order finite-difference method for simulation isotropic turbulence flow

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Abstract: The work deals with the modeling of turbulent energy using two different methods: finite-difference and spectral methods. To simulate the turbulent process the filtered three-dimensional non-stationary Navier-Stokes equation is used. The problem is solved by using hybrid methods, where the equation of motion is solved by using finite-difference methods in combination with cyclic penta-diagonal matrix, which allowed to reach high order of accuracy and to simulate turbulence decay at Reynolds number Re=300 for comparing with theoretical solution of Taylor and Green. The spectral method is used for solution of Poisson equation, which is makes it possible to gain the time. For validation of the given algorithm we solved the classical problem of Taylor and Green modeling of isotropic turbulence flow. Where at the work of Taylor and Green [1], the problem is solved analytically and defined all the turbulence characteristics. In the results of our simulation we have compared the turbulence characteristics with analytical solution: the change of turbulent kinetic energy, dissipation energy over the time and longitudinal and transverse one-dimensional spectra are defined. In this paper a three-dimensional non-stationary Navier-Stokes equation is solved for simulation of isotropic turbulence flow. For solving Navier-Stokes equation, we use a splitting scheme by physical parameters that consist of three stages. At the first stage, the NavierStokes equation is solved, without taking pressure into account. For approximation of the convective and diffusion terms of the intermediate velocity field finite-difference methods in combination with cyclic penta-diagonal matrix is used [2], which allowed to increase the order of accuracy in time and in space $O(t^3, h^4)$ without changing the amount of points. At the second stage the Poisson equation is solved, which is satisfies the continuity equation with considering the velocity field from the first stage. For solving the Poisson equation we use spectral method in combination with Fourier transform. The obtained pressure field with using fast Fourier transform is translated from the phase space to physical and used at the third stage for the recallculate of the final velocity field [3].

Keywords: finite-difference method, spectral method, Poisson equation, cyclic penta-diagonal matrix

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