Criterion of the boundedness of a fractional integration type operator with variable upper limit in weighted Lebesgue spaces

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Abstract: Let $0 < p, q < \infty$, $I = (a, b)$, $-\infty \le a < b \le \infty$, $0 < \alpha < 1$, $\frac{1}{p} + \frac{1}{p'} = 1$, $R_+ = (0, \infty)$, $W : I \to R$ is a nondecreasing, locally absolutely continuous function and $W(a) = \lim\limits_{t \to a^+} W(t) > -\infty$. $\frac{dW(t)}{dt} = w(t)$, $v : I \to R$ is a non-negative locally integrable function on I and $\varphi : I \to I$ is a strictly increasing locally absolutely continuous function with the properties $\lim\limits_{x \to a^+} \varphi(x) = a$, $\lim\limits_{x \to b^-} \varphi(x) = b$, $\varphi(x) \le x$, $\forall x \in I$. Consider the operator in a form $K_{\alpha,\varphi} f(x) = \int\limits_a^{\varphi(x)} \frac{f(s)w(s)ds}{(W(x)-W(s))^{1-\alpha}}$, $x \in I$ from $L_{p,w} = L_{p,w}(I)$ to $L_{q,v} = L_{q,v}(I)$, where $L_{p,w}$ is a space of measurable functions $f : I \to R$ for which the functional $\|f\|_{p,w} = \left(\int\limits_a^b |f(x)|^p w(x) dx \right)^{\frac{1}{p}}$, $0 < p < \infty$ is finite. In the case $\varphi(x) \equiv x$ the operator $K_{\alpha,\varphi}$ is studied in papers [1], [2].

Theorem. The operator $K_{\alpha,\varphi}$ is bounded from $L_{p,w}$ to $L_{q,v}$ if and only if

1) $A = \sup\limits_{a<z<b} \left(\int\limits_z^b W^{q(\alpha-1)}(x) v(x) dx \right)^{\frac{1}{q}} W^{\frac{1}{p'}}(\varphi(x)) < \infty$, for $1 < p \le q < \infty$ and $\frac{1}{p} < \alpha < 1$, where $\|K_{\alpha,\varphi}\| \approx A$;

2) $$B = \left(\int\limits_a^b \left(\int\limits_t^b W^{q(\alpha-1)}(x) v(x) dx \right)^{\frac{q}{p-q}} W^{\frac{q(p-1)}{p-q}}(\varphi(t)) \frac{v(t)dt}{W^{q(1-\alpha)}(t)} \right)^{\frac{p-q}{pq}} < \infty,$$
when $0 < q < p < \infty$, $p > \frac{1}{\alpha}$, $0 < \alpha < 1$ and $\|K_{\alpha,\varphi}\| \approx B$, where $\|K_{\alpha,\varphi}\|$ is a norm of the operator $K_{\alpha,\varphi} : L_{p,w} \to L_{q,v}$.

Keywords: fractional integration operator, weighted estimate.

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References

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