Weighted estimates for the integral operator with a logarithmic singularity

Akbota Abylayeva
L.N. Gumilyov Eurasian National University,
010008 Astana, Satpayev str., 2, Kazakhstan
E-mail: abylayeva_b@mail.ru

Abstract: Let $0 < q < \infty$, $1 < p < \infty$, $\frac{1}{p} + \frac{1}{p'} = 1$, $R_+ = (0, \infty)$, $U : R_+ \to R$ and $V : R_+ \to R$ are wheighted functions, i.e. non-negative measurable functions on R_+.

Since the 70-s of the last century the weighted estimate of the form

$$\|UKf\|_q \leq C\|Vf\|_p \tag{1}$$

intensively studied in the world mathematical literature for different classes of the operators K, where $\|\cdot\|_p$ is the usual norm of the space $L_p \equiv L_p(R)$. Some directions of the research of the estimate (1) until 2000 year for the integral operators are summarized in the books [1], [2], [3]. In the paper [4] the estimate (1) was studied for some classes of the operator Kf and also a full description of weights U and V was given there.

In this paper we study the estimate (1) for the singular operator in the form $Kf(x) = \int\limits_0^x \ln \frac{x}{x-s} f(s)ds$, when $V(x) = x^{1-\gamma}$, $\gamma > 0$.

Theorem. *Let $1 < p \leq q < \infty$, $\gamma > \frac{1}{p}$. The inequality (1) holds if and only if*

$$A = \sup_{x>0} x^{\gamma + \frac{1}{p'}} \left(\int\limits_x^\infty U^q(t) t^{-q} dt \right)^{\frac{1}{q}} < \infty,$$

wherein $C \approx A$, where C is the best constant (1).

Keywords: integral operator with logarithmic singularity, weighted estimate.

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References

[1] Opic, B., Kufner, A., "Hardy-type inequalities", *Pitman Research Notes in Math. Series. Longman Scientific and Technical*, Horlow, 1990.

[2] Kufner, A., Persson, L-E., "Integral inegualities with weights", Prague. 2000. -.211.

[3] Kufner, A., Maligranda, L., Persson, L.-E."The Hardy inequality -about its history and some related results, *University of West Bohemia, Plzen*, p. 152, 2007.

[4] Oinarov, R., "Boundedness and compactness of integral operators Voltaire types", *Sib. Math. J.*, Vol. 48, No. 5, pp. 1100-1115, 2007. (in Russian).