The first regularized trace of integro-differential Sturm-Liouville operator on the segment with punctured points at integral perturbation of transmission conditions

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Abstract: The report is devoted to calculating a first regularized trace of one integro-differential operator with the main part of the Sturm-Liouville type on a segment with punctured points at integral perturbation of "transmission" conditions. The Sturm-Liouville operator

$$-y''(x) + q(x)y(x) + \gamma \int_0^\pi y(t)dt = \lambda y(x)$$

given on the segments $\frac{\pi}{n}(k-1) < x < \frac{\pi}{n}k,\ k = \overline{1,n};\ n \geq 2$ is considered. Boundary conditions of the Dirichlet type:

$$y(0) = 0, \quad y(\pi) = 0$$

are given on the left-hand and right-hand ends of the segment $[0, \pi]$. The functions continuous on $[0, \pi]$, the first derivatives of which have jumps at the points $x = \frac{\pi}{n}k$, are solutions. The value of jumps is expressed by the formula

$$y'\left(\frac{\pi k}{n} - 0\right) = y'\left(\frac{\pi k}{n} + 0\right) - \beta_k \int_0^\pi y(t)dt, \quad k = \overline{1, n-1}.$$

The basic result of the report is the exact formula of the first regularized trace of the considered differential operator.

Keywords: spectral problem, first regularized trace, integro-differential operator, inner-boundary condition

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