On a difference scheme for nonlocal heat transfer boundary-value problem

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Abstract: In this paper, the family of boundary value problems for heat equation and finite difference schemes approximating these problems are considered. The peculiarity of the initial-boundary value problems is a special choice of the boundary conditions, which are not strongly regular. The corresponding difference schemes do not have the property of self-adjoint.

The paper proposes a new method of solving non-local problems for the heat equation

$$u_t(x,t) - u_{xx}(x,t) = f(x,t), \tag{1}$$

satisfying the initial condition

$$u(x,0) = \phi(x), \quad 0 \le x \le 1, \tag{2}$$

and the boundary conditions

$$\begin{cases} u_x(0,t) - u_x(1,t) + au(0,t) + bu(1,t) = 0, \\ cu(0,t) + (1+c)u(1,t) = 0. \end{cases} \tag{3}$$

with finite difference method. The main important feature of these problems is their non-self-adjointness. This non-self-adjointness causes major difficulties in their analytical and numerical solving. The problems, which boundary conditions do not possess strong regularity, are less studied. The scope of study of the paper justifies possibility of building a stable difference scheme with weights for abovementioned type of problems.

Keywords: heat equation, initial boundary problems, numerical method, difference scheme.

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