Estimations of the best $M-$ term approximations of functions in the Lorentz space with constructive methods

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Abstract: V.N. Temlyakov [1, 2] developed a constructive method of estimation of the best $n-$ term approximations functions from Nikol'skii-Besov's classes $S^{\bar{r}}_{p,\tau}B$ in the space $L_q(I^m)$, $1 < p < q < \infty$. In talk considers $L^*_{\bar{q},\bar{\theta}}(I^m)$ – the Lorentz space of periodic functions of many variables with the anisotropic norm $\|f\|^*_{\bar{q},\bar{\theta}}$. $e_n(f)_{\bar{q},\bar{\theta}}$ - the best n-term approximation of f. $e_n(F)_{\bar{q},\bar{\theta}} = \sup_{f\in F} e_n(f)_{\bar{q},\bar{\theta}}$.

Suppose $\rho(\bar{s}) = \left\{\overline{k} = (k_1, ..., k_m) \in \mathbb{Z}^m : \quad 2^{s_j-1} \leq |k_j| < 2^{s_j}, j = 1, ..., m\right\}$, $\delta_{\bar{s}}(f, \overline{x}) = \sum\limits_{\overline{n}\in\rho(\bar{s})} a_{\overline{n}}(f)\, e^{i\langle\overline{n},\overline{x}\rangle}$, where $\langle\bar{y}, \bar{x}\rangle = \sum\limits_{j=1}^{m} y_j x_j$, $s_j = 1, 2, ...$ For a function $f \in L_1(I^m)$ suppose $f_l(\bar{x}) = \sum\limits_{\bar{s}:\langle\bar{s},\bar{\gamma'}\rangle=l} \delta_{\bar{s}}(f, \bar{x})$, $\bar{x} \in I^m$, $l \in \mathbb{N}\cap\{0\} = \mathbb{N}_0$, where $\bar{\gamma}' = (\gamma_1^{'}, ..., \gamma_m^{'})$, $\gamma_j^{'} = (\frac{1}{q_j} - \frac{1}{p}))/(\frac{1}{q_1} - \frac{1}{p})$. We consider the class $W^{a,b}_{\bar{q},\theta} = \left\{f \in L_1(I^m) : \|f_l\|^*_{\bar{q},\theta} \leq 2^{-la}\overline{l}^{(\nu-1)b}\right\}$, where $a > 0, b > 0$, $\bar{l} = \max\{1, l\}$ and let us introduce the notation $\|f\|_{W^{a,b}_{\bar{q},\theta}} = \sup\limits_{l\in\mathbb{N}_0} \|f_l\|^*_{\bar{q},\theta} 2^{la}\bar{l}^{-(\nu-1)b}$. In the case $q_1 = ... = q_m = \theta$ this class considered V.N. Temlyakov [1]. The main results:

Theorem. Let $1 < q_m \leq ... \leq q_{\nu+1} < q_\nu = ... = q_1 < p < 2$, $1 < \theta < p$.

1. If $a > \frac{1}{q_1} + \frac{1}{\theta} - \frac{2}{p}$, then $e_n\big(W^{a,b}_{\bar{q},\theta}\big)_p \asymp n^{-a+\frac{1}{q_1}-\frac{1}{p}}(\log n)^{(\nu-1)(a+b+\frac{2}{p}-\frac{1}{q_1}-\frac{1}{\theta})}$.
2. If $\frac{1}{q_1} - \frac{1}{p} < a < \frac{1}{q_1} + \frac{1}{\theta} - \frac{2}{p}$, then $e_n\big(W^{a,b}_{\bar{q},\theta}\big)_p \asymp n^{-a+\frac{1}{q_1}-\frac{1}{p}}(\log n)^{(\nu-1)b}$.
3. If $a = \frac{1}{q_1} + \frac{1}{\theta} - \frac{2}{p}$, then $e_n\big(W^{a,b}_{\bar{q},\theta}\big)_p \asymp n^{-(\frac{1}{\theta}-\frac{1}{p})}(\log n)^{(\nu-1)b}(\log\log n)^{\frac{1}{\theta}}$.

Keywords: Lorentz space, M-term approximations, constructive method

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References

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