The maximum principle for the Navier-Stokes equations

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Abstract: New connections were established between extreme values of the velocity, the density of kinetic energy and the pressure of the problem for the Navier-Stokes equations in domain $Q = (0, T] \times \Omega$:

$$\frac{\partial \mathbf{U}}{\partial t} - \mu\Delta\mathbf{U} + (\mathbf{U}, \nabla)\mathbf{U} + \nabla P = \mathbf{f},\ \mathrm{div}\mathbf{U} = 0, \tag{1}$$

$$\mathbf{U}(0, \mathbf{x}) = \mathbf{\Phi}(\mathbf{x}),\ \mathbf{x} \in \Omega \subset R_3;\ \mathbf{U}(t, \mathbf{x}) = 0,\ \mathbf{x} \in \partial\Omega. \tag{2}$$

Validity of the maximum principle was shown for problem (1), (2) using these connections:

$$\|\mathbf{U}\|_{\mathbf{C}(\bar{Q})} \leq \|\mathbf{\Phi}\|_{\mathbf{C}(\bar{\Omega})} + T\|\mathbf{f}\|_{\mathbf{C}(\bar{Q})} \equiv const,\ \forall T < \infty.$$

On the basis of which the selected space to prove the uniqueness of the weak and the existence of strong solutions to the problem Navier-Stokes equations for the whole time.

Keywords: Extreme values vector of velocity, density of kinetic energy, pressure, the maximum principle for the Navier-Stokes equations

2010 Mathematics Subject Classification: 35Q30, 35Q35

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