The correctness Dirichlet problem in a cylindrical domain for three-dimensional elliptic equations with type and order extinction

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Abstract: Correct formulation of boundary value problems in the plane for elliptic equations by the method of the theory of analytic complex variable functions is well studied. In the study of similar issues, when the number of independent variables is greater than two, there are difficulties of a fundamental nature. Very attractive and convenient method of singular integral equations loses its power because of absence of any comprehensive theory of multidimensional singular integral equations. Previously correctness of the Dirichlet problems for degenerate three-dimensional elliptic equations has been established. Importance of study of boundary value problems for the elliptic equations with degeneration of order and type was noticed by A.V. Bitsadze. Let $D_\beta-$ be a cylindrical domain of the Euclidian space E_3 of points (x_1, x_2, t), bounded by the cylinder $\Gamma = \{(x,t) : |x \,|= 1\}$, the planes $t = \beta > 0$ and $t = 0$, where $|x\ |-$ is the length of the vector $x = (x_1, x_2)$. Let us denote the parts of these surfaces forming the boundary ∂D_β of the domain D_β as $\Gamma_\beta, S_\beta, S_o$ respectively. Let us consider three-dimensional hyperbolic equations with type and order confluence in the domain D_β :

$$\sum_{i=1}^{2} k_i(t)u_{x_ix_i} - k_3(t)u_{tt} + \sum_{i=1}^{2} a_i(x,t)u_{x_i} + b(x,t)u_t + c(x,t)u = 0, \qquad (1)$$

where $k_i(t) > 0$ when $t > 0$ and they vanish when $t = 0$, $k_i(t) \in C([0,\beta]) \cap C^2((0,\beta)), i = 1,2,3$. Equation (1) is elliptic if and along the plane there is a degeneration of its type and order. As a multi-dimensional Dirichlet problem, we consider

Problem 1. Find the solutions of the equation (1) in the domain D_β from the class $C(\bar{D}_\beta) \cap C^2(D_\beta)$, which satisfy the boundary value conditions

$$u|_{S_\beta} = \varphi(x), u|_{\Gamma_\beta} = \psi(x,t), u|_{S_o} = \tau(x).$$

For smooth coefficients of the equation (1) and certain conditions on the boundary data unique solvability is proved, and an explicit form of classical solution of the problem D is obtained.

Keywords: correctness, problem, equation, type and order extinction

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