On solvability of a class of nonlinear two-point boundary value problems

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Abstract: We study the solvability of a class of nonlinear two-point boundary value problems for systems of ordinary second-order differential equations on the plane. On the basis of properties of the leading nonlinear terms, we prove a criterion for the solvability of boundary value problems under arbitrary perturbations in a given set by using methods for the computation of the winding number of vector fields.

In this work, we study the solvability of nonlinear two-point boundary value problems of the form

$$(1) \qquad z''(t) = \overline{z'(t)}^m + f(t, z(t), z'(t)), \quad z = x + iy \in \mathbf{C}, \quad 0 < t < 1,$$

$$(2) \qquad z'(0) = A_0(z(0)) + h_0(z), \quad z'(1) = A_1(z(0)) + h_1(z).$$

Here m is an integer larger than unity, $\mathbf{C}$ is the complex plane, $\overline{z} = x - iy$, and the mappings $f : [0,1] \times \mathbf{C}^2 \mapsto \mathbf{C}$ and $A_0, A_1 : \mathbf{C} \mapsto \mathbf{C}$ are continuous and satisfy the conditions:

$$(3) \qquad A_j(\lambda z) \equiv \lambda A_j(z) \quad \text{for any } \lambda \geq 0, j = 0, 1,$$

$$(4) \qquad \max_{0 \leq t \leq 1} |f(t, z, w|(|z| + |w|)^{-m} \to 0 \quad \text{as } |z| + |w| \to \infty.$$

The mappings h_0 and h_1 act continuously from $C^1([0,1]; \mathbf{C})$ into $\mathbf{C}$, where $C^1([0,1]; \mathbf{C})$ is the space of complex-valued functions continuously differentiable on the interval $[0,1]$, and satisfy the conditions

$$(5) \qquad |h_j(z)| / ||z||_{C^1} \to 0 \quad \text{as } ||z||_{C^1} \to \infty, j = 0, 1.$$

For this class of boundary value problems, we study the solvability on the basis of an a priori estimate of solutions with the use of methods for the computation of the winding number of vector fields [1]. This problem was earlier considered in [2, 3, 4] and is of interest from the viewpoint of the application of methods of nonlinear analysis and the development of investigation methods for nonlinear two-point boundary value problems.

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