On spectral problems for loaded two-dimension Laplace operator

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Abstract: In domain $Q = \{x, y : -\pi < x < \pi, \ -\pi < y < \pi\}$ we consider following two spectral problems:

$$\begin{cases} -\triangle\varphi(x,y) + \alpha\varphi(0,y) = \lambda\varphi(x,y), \ \ \{x,y\} \in Q, \\ \dfrac{\partial^j\varphi(-\pi,y)}{\partial x^j} = \dfrac{\partial^j\varphi(\pi,y)}{\partial x^j}, \ \ \dfrac{\partial^j\varphi(x,-\pi)}{\partial y^j} = \dfrac{\partial^j\varphi(x,\pi)}{\partial y^j}, \ \ j = 0, \ 1; \end{cases} \tag{1}$$

$$\begin{cases} -\triangle\varphi(x,y) + \alpha\varphi(0,y) + \beta\varphi(x,0) = \lambda\varphi(x,y), \ \ \{x,y\} \in Q, \\ \dfrac{\partial^j\varphi(-\pi,y)}{\partial x^j} = \dfrac{\partial^j\varphi(\pi,y)}{\partial x^j}, \ \ \dfrac{\partial^j\varphi(x,-\pi)}{\partial y^j} = \dfrac{\partial^j\varphi(x,\pi)}{\partial y^j}, \ \ j = 0, \ 1; \end{cases} \tag{2}$$

where Δ is Laplace operator, α, $\beta \in \mathbb{C}$ are given complex numbers, $\lambda \in \mathbb{C}$ is a spectral parameter.

Note that the need to investigate of spectral problems (1) and (2) arise in solving of the stabilization problem for a loaded heat equation with help of the boundary control actions.

Depending on the values of α and β, we give a complete description of the eigenvalues, eigenfunctions and associated functions of the set of spectral problems (1) and (2).

Keywords: Loaded Laplace operator, spectrum, eigenfunction

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References

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