Inverse problem for the Verhulst equation of limited population growth with discrete experiment data

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Abstract: Verhulst limited growth model with unknown parameters of growth is considered. These parameters are defined by discrete experiment data. This inverse problem is solved with using gradient method with interpolation of data and without it [1, 2]. Approximation of the delta-function is used for the latter case. As an example the bacteria population *E.coli* is considered.

We consider the evolution of biological species in limited habitat. The given system is described by Verhulst equation. The coefficients of the equations are unknown. We would like to determine it from an inverse problem by using the results of the experiment. The data are distributed discretely. The inverse problem is transformed to an extremum one. This problem is solved by gradient method. The gradient of the given functional depends on the solution of the adjoint system. The adjoint equation includes delta-functions due to the discreteness of the data. This difficulty can be leave out by the interpolation of the data. This inverse problem is solved by using gradient method with interpolation of data and without it. The delta-function is approximated by Gauss formula [3]. The calculation are realized for the exact values of data and with noised data. The calculation accuracy is high enough for both cases and algorithms. We find also the parameters of growth for bacteria population E.coli by real experiment of Scientific Centre of Anti-infectious Drugs (Almaty).

Keywords: Verhulst equation, inverse problem, delta-function

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