Binarity for almost ω-categorical quite o-minimal theories

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Abstract: The present lecture deals with the notion of *weak o-minimality*, which initially deeply studied in [1]. A subset A of a linearly ordered structure M is *convex* if for any $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \ldots \rangle$ such that any definable (with parameters) subset of the structure M is a finite union of convex sets in M.

In the following definitions (introduced in [2] and [3] respectively) we assume that M is a weakly o-minimal structure, $A \subseteq M$, M is $|A|^+$-saturated, and $p, q \in S_1(A)$ are non-algebraic types. We say that p is not *weakly orthogonal* to q ($p \not\perp^w q$) if there are an A-definable formula $H(x, y)$, $\alpha \in p(M)$, and $\beta_1, \beta_2 \in q(M)$ such that $\beta_1 \in H(M, \alpha)$ and $\beta_2 \notin H(M, \alpha)$. We say that p is not *quite orthogonal* to q ($p \not\perp^q q$) if there is an A-definable bijection $f : p(M) \to q(M)$. We say that a weakly o-minimal theory is *quite o-minimal* if the relations of weak and quite orthogonality for 1-types coincide.

Almost ω-categoricity has been introduced in [4] and studied in [5].

Theorem. Any almost ω-categorical quite o-minimal theory is binary.

Keywords: almost ω-categoricity, weak o-minimality, quite o-minimality, binary theory

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