Weighted inequality of Hardy type

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Abstract: This work is an extension of the results of article. The necessary and sufficient conditions on weight functions ρ, ν, r are obtained, for which the following inequality holds true

$$\|rf\|_q = c\left(\|\rho f'\|_p + \|\nu f\|_p\right) \tag{1}$$

for $p = q = \infty$. And for the smallest constant two-sided estimates are found, which are of the same order. The criterion of compactness of embedding operator, that corresponds to (1) is provided. Let us note that (1) becomes a generalized Hardy inequality (in differential form) as $\nu \equiv 0$. For $\rho \equiv 1, \quad 1 = p \leq q < \infty$, the inequality (1) was examined by K.T. Mynbayev, for $\rho \equiv 1, \quad p = q = 2$, by E.T. Sawyer, for $1 < p \leq q < \infty$ by M. Otelbaev and R.Oynarov, and for $1 \leq p < q < \infty, \quad 1 \leq q < p < \infty$, in general situations by R. Oynarov. Similar inequalities for matrix operators are considered in the works of J. Tasmagambetova and A. Temirkhanova.

We introduce the following parameters describing the local behavior of the weighting functions:

$$\omega(x,y) = \sup\left\{d > 0 : \int_{x-d}^{x} \rho^{-1}(\tau)\, d\tau \leq \int_{x}^{x+y} \rho^{-1}(\tau)\, d\tau, (x-d, x+y] \subset J\right\},$$

$$d^+(x) = \sup\left\{d > 0 : \sup_{x-\omega(x,d) \leq t \leq x+d} \upsilon(t) \int_{x-\omega(x,d)}^{x+d} \rho^{-1}(\tau)\, d\tau \leq 1, [x, x+d) \subset J\right\},$$

$$d^-(x) = \omega\left(x, d^+(x)\right).$$

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