On the compactness of set in generalized Morrey Spaces

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Abstract: In this paper, we present sufficient conditions for strongly pre-compact sets in the generalized Morrey spaces M_p^w.

Let $1 \le p \le \infty$, $w(x)$ measurable non-negative function on $(0,\infty)$. Generalized Morrey space $M_p^{w(R)}$ is defined as the set of functions $f \in L_1^{loc}(R)$ with a finite quasi-norm: $\|f\|_{M_p^w} \equiv \sup\limits_{t\in R^n, r>0} w(r) \left(\int\limits_{B(t,r)} |f(x)|^p \, dx \right)^{\frac{1}{p}} < \infty$, where $B(t,r)$ is a ball with center at the point t and with radius r. For $f \in L_1^{loc}(R)$ and $\alpha > 0$ suppose $(M_\alpha f)(x) = \frac{1}{|B(x,\alpha)|} \int\limits_{B(x,\alpha)} f(y)dy$. We denote by Ω_p the set of all functions which are non-negative , measurable on Ω_p and for some $t_1, t_2 > 0$: $\sup\limits_{t_1 \le r < \infty} (w(r)) < \infty$, $\sup\limits_{0<r\le t_2} \left(w(r) r^{\frac{1}{p}} \right) < \infty$.

Theorem Suppose that $1 \le p \le \infty$ and $w \in \Omega_p$. Suppose the subset S in M_p^w satisfies the following conditions:

$$\sup_{f\in S} \|f\|_{M_p^w} < \infty, \ \lim_{u\to 0} \sup_{f\in S} \|f(\cdot + u) - f(\cdot)\|_{M_p^w} = 0, \ \lim_{r\to\infty} \sup_{f\in S} \left\| f \chi_{C_{B(0,r)}} \right\|_{M_p^w} = 0.$$

Then S is strongly pre-compact set in $M_p^w(R)$.

From the proved theorem for the case of $w(r) = r^{-\lambda}$ follows a well-known result for the Morrey space M_p^λ [3] and in the case of $\lambda = 0$ this is well-known Frechet-Kolmogorov theorem.

Keywords: Generalized Morrey spaces, compactness

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References

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