The stabilization rate of a solution to the Cauchy problem for parabolic equation with lower order coefficients

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Abstract: In the half-space $D = R^N \times (0, \infty), N \geqslant 3$, consider the Cauchy problem for a non divergent parabolic equation.

$$L_1 u \equiv Lu + (b, \nabla u) + c(x,t)u - u_t = 0, \quad (x,t) \in D, \tag{1}$$

$$u(x,0) = u_0(x), \quad x \in R^N, \tag{2}$$

where
$$Lu = \sum_{i,k=1}^{N} a_{ik}(x,t) u_{x_i x_k}, \quad (b, \nabla u) = \sum_{i=1}^{N} b_i(x,t) u_{x_i}. \tag{3}$$

In this report we study sufficient conditions on the lower order coefficients of a parabolic equation guaranteeing the power rate of the uniform stabilization to zero of the solution to the Cauchy problem on every compact in R^N and for any bounded initial function.

We assume that following conditions are satisfied:

$$\exists k_0^2 > 0: \ k_0^2 = \inf_{D, |\xi|=1} \sum_{i,k=1}^{N} a_{ik}(x,t)\xi_i \xi_k; \ \exists k_1^2 > 0: \ k_1^2 = \sup_{D, |\xi|=1} \sum_{i,k=1}^{N} a_{ik}(x,t)\xi_i \xi_k.$$

The coefficients $b_i(x,t) (i = 1, ..., N)$ satisfy *condition* (B); i.e., there exist constant $B > 0$ such as

$$\sup_{(x,t)\in D} (1+r) \sum_{i=1}^{N} |b_i(x,t)| = B < k_0^2 N, \quad \text{where } r = \sqrt{x_1^2 + \cdots + x_N^2}.$$

The coefficient $c(x,t)$ satisfies *condition (C)*, i.e. exist $\alpha > 0$ for which

$$c(x,t) \leqslant a_\alpha(r) = -\alpha^2 \min(1, r^{-2}).$$

Let $\lambda_1(\alpha) = (2 - S + \sqrt{D_1})/2, \ D_1 = (2-S)^2 + 4\overline{\alpha}^2, \ \overline{\alpha} = \alpha / k_1.$

Theorem. *Suppose that* (B) *and* (C) *with*

$$\alpha^2 > k_1^2 (S-1), \ S = \left(k_1^2 (N-1) + k_0^2 + B\right)/k_0^2,$$

hold. Then $\lim\limits_{t\to\infty} t^{\nu_n} u(x,t) = 0$, $\nu_n = \lambda_1(\alpha)/(2 + \frac{1}{n})$ *for any* $n = 1, 2, ...$, *uniformly with respect to* x *on any compact* K *in* R^N.

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