On the solvability of a nonlocal boundary value problem for the Laplace operator with opposite flows at the part of the boundary

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Abstract: In the present paper we investigate a nonlocal boundary problem for the Laplace equation in a half-disk, with opposite flows at the part of the boundary. Our goal is to find a function $u(r,\theta) \in C^0(\bar{D}) \cap C^2(D)$ satisfying in D the equation

$$\Delta u = 0$$

with the boundary conditions

$$u(1,\theta) = f(\theta),\ 0 \leq \theta \leq \pi,$$

$$u(r,0) = 0, r \in [0,1],$$

$$\frac{\partial u}{\partial \theta}(r,0) = -\frac{\partial u}{\partial \theta}(r,\pi) + \alpha u(r,\pi),\ r \in (0,1)$$

where $D = \{(r,\theta) : \quad 0 < r < 1, 0 < \theta < \pi\}$; $\alpha < 0$; $f(\theta) \in C^2[0,\pi]$, $f(0) = 0$, $f'(0) = -f'(\pi) + \alpha f(\pi)$.

The difference of this problem is the impossibility of direct applying of the Fourier method (separation of variables). Because the corresponding spectral problem for the ordinary differential equation has the system of eigenfunctions not forming a basis. Throughout this note we mainly use techniques from our work [1]. A special system of functions based on these eigenfunctions is constructed. This system has already formed the basis. This new basis is used for solving of the nonlocal boundary value problem. The existence and the uniqueness of the classical solution of the problem are proved.

Keywords: Laplace equation, basis, eigenfunctions, nonlocal boundary value problem

2010 Mathematics Subject Classification: 33C10, 34B30, 35J, 35P10

References

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