About the correctness to some nonlocal boundary value problem for the equation of the mixed type of the first kind and the second order in space

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Abstract: As it is known the Dirichlet problem for the equation of the mixed type of the first sort is incorrect. Naturally there is a problem: whether it is impossible to substitute statements of the problem of Dirichlet other conditions enveloping all boundary which ensure a problem correctness? For the first time, such problems have been offered and studied in the works T.S.Kalmenov's. In the present work for the equation of the mixed type of the first kind, the second order in space unambiguously solvability and smoothness of the generalized solution, of some nonlocal boundary value problem with constant coefficients from Sobolev spaces is studied.

Let $\Omega = \prod\limits_{i=1}^{n} (\alpha_i, \beta_i) \in R^n$. $n-$ a measured parallelepiped.

In the area $Q = \Omega \times (0, T)$ we consider a differential equation of the second order

$$Lu = K(x)\, u_{tt} - \sum_{i,j=1}^{n} \frac{\partial}{\partial\, x_i} \left(a_{i,j}(x)\, u_{x_j} \right) + \alpha\,(x,t)\, u_t + c(x,t)\, u = f(x,t), \quad (1)$$

where, $x_1 K(x) > 0$ at $x_1 \neq 0$ and thus $x_1 \in (\alpha_1, \beta_1)$, $\alpha_1 < 0 < \beta_1$.

We assume, that coefficients of equation (1) -are smooth enough functions and let the condition following is satisfied: $a_{i,j}\xi_i\xi_j \geq a_0 |\xi|^2$, where,

$a_{i,j}(x) = a_{j,i}(x)$; $a_0 - const > 0$, $\xi \in R^n$; $|\xi|^2 = \sum\limits_{i=1}^{n} \xi_i^2$.

Problem. To find a generalized solution of equation (1) from the Sobolev space $W\,^l_2(Q)$, ($2 \leq l$- is integer), satisfying to nonlocal boundary conditions

$$\gamma D_t^p\, u|_{t=0} = D_t^p\, u|_{t=T} \quad (2)$$

$$\eta D_{x_i}^p\, u|_{x_i=\alpha_i} = D_{x_i}^p\, u|_{x_i=\beta_i}, \quad (3)$$

at $p = 0, 1$; here γ *and* $\eta - const \neq 0$, where $D_t^p u = \frac{\partial^p u}{\partial t^p}$ $D\,^0_t u = u$.

Keywords: second order mixed type equation of the first kind,nonlocal boundary value problem with constant coefficients, Sobolev spaces,unique solvability,existence of solution, smoothness of the generalized solution.

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