On the properties of operators with the semi-power type lacunas

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Abstract: For linear matrix operators

$$u_N(f,\Lambda,X)=\frac{a_0}{2}+\sum_{k=1}^{N}\lambda_k^{(n)}(a_k\cos kx+b_k\sin kx),$$

(a_k, b_k are Fourier coefficients of $f(x)$), constructed with respect to subsequences of the Fourier sums S_{n_k} with the lacunas $n_k=[\frac{k^\gamma}{\ln^\beta(k+1)}],\ \ k=1,2,...,N,$ a sufficient condition for boundedness of norms $\|u_N\|_{C_{2\pi}}$

$$(1)\qquad \|u_N\|\le c\{n_N^{\frac{1}{\gamma}}\sum_{k=0}^{N}(\Delta\lambda_k)^2\ln^{\beta/\gamma}(k+1)[1+(N-k)^{1-1/\gamma}k^{1/\gamma-1}]\},$$

is given. N is the number of non-zero differences $\Delta\lambda_k=\lambda_{n_k}-\lambda_{n_k+1},\ c>0,\ c=const.$ $[y]$ is the integer part of y. $\beta\ge 0,\ \gamma\ge 1$. $\lambda_k^n\equiv\lambda_k$ are limited multipliers of summation.

Using (1) approximative properties of the discrete analog of biharmonic operators $D(f,r,x)$ are investigated. In particular the precise estimate

$$(2)\ \sup_{f\in C^1}\|f(x)-D(f,r,x)\|_{C_{2\pi}}=O\big((\gamma-\beta)\sqrt{(1-r)\ln^{\beta/\gamma}\frac{1}{1-r}}\big),\ r\to 1-.$$

is obtained in the class of differentiable functions C^1.

If $\gamma=1,\ \beta=0$ then (2) coincides with the estimate of the work [1].

Keywords: semi-power type lacunas, biharmonic operator

2010 Mathematics Subject Classification: 42A24

References

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