On spectrum of perturbed two-dimensional harmonic oscillator in a strip

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Abstract:

Let $\Pi = \{(x,y), x \in \mathbb{R}, 0 \le y \le \pi\}$, $L_0 = -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + x^2$. We denote by V the operator of multiplication by bounded measurable real-valued function $V(x,y)$, which is finite on the variable x. That is, there exists $r > 0$ with the property: $V(x,y) \equiv 0$, for any x, $|x| \ge r$. In our further considerations such functions $V(x,y)$ are said to be *finite in the strip* Π. We denote $\operatorname{supp} V$ by $\Pi_0(r)$. Here, we deal with the Dirichlet problem operator $L = L_0 + V$ in the space $L^2(\Pi)$. Notice that the title of the paper is just prompted by the possibility to consider the expression $(\widetilde{V}(x,y) + y^2)$, $y \in [0,\pi]$, instead of $V(x,y)$, without loss of generality.

Let $P_s^{(1)}$ $P_l^{(2)}$ be orthogonal projectors onto eigensubspaces of one-dimensional Laplace operators of Dirichlet problem and harmonic oscillator corresponding to the eigenvalues s^2, $s = 1, 2...$, and $2l+1$, $l = 0, 1, 2...$, correspondingly. That is, $P_s^{(1)} f = (f, f_s) f_s$, $f_s(y) = \sqrt{\frac{2}{\pi}} \sin sy$ $P_l^{(2)} f = (f, \varphi_l)\varphi_l$, $\varphi_l(x) = (2^l l! \sqrt{\pi})^{-\frac{1}{2}} e^{-\frac{x2}{2}} H_l(x)$, where $H_l(x)$ are Hermitian polynomials.

Then, we have the following assertion on the eigenvalues $\lambda_{sl} = s^2 + 2l + 1$, $s = 1, 2, ..., l = 0, 1, 2, ...$ of the operator L_0

Theorem 1. The spectrum of the operator L_0 consists of the eigenvalues $\lambda_n = n$, $n \in \mathbb{N} \setminus \{1, 3\}$, with multiplicities ν_n

$$\nu_n = \begin{cases} [\frac{\sqrt{n}}{2}], & \lambda_n \in \left((2[\frac{\sqrt{n}}{2}])^2; (2[\frac{\sqrt{n}}{2}]+1)^2\right) \\ [\frac{\sqrt{n}}{2}] + \frac{(-1)^n+1}{2}, & \lambda_n \in \left((2[\frac{\sqrt{n}}{2}]+1)^2; (2[\frac{\sqrt{n}}{2}]+2)^2\right). \end{cases}$$

The corresponding projector operators are $P_n = \sum_{s=1}^{\nu_n} P_s^{(1)} \otimes P^{(2)}_{(\frac{n}{2} - \frac{s^2+1}{2})}$.

For each $n \in \mathbb{N} \setminus \{1, 3\}$ we denote by $\mu_s^{(n)}$, $s = 1, \ldots, \nu_n$, the eigenvalues of the operator L corresponding to the eigenvalue λ_n of the operator L_0. We

have established that there exists $d > 0$ with the property: for $n >> 1$ the inequalities $\left|\mu_s^{(n)} - \lambda_n\right| \leq dn^{-\frac{1}{2}}$ hold for any $s = 1, \ldots, \nu_n$.

Using the method of our paper [1] we get our main result:

Theorem 2. Assume that $V(x,y) \in C^4(\Pi_0(r))$. Then,

$$\sum_{k\in\mathbb{N}\setminus\{1,3\}} \left[\sum_{s=1}^{\nu_k} \left(\lambda_k - \mu_s^{(k)}\right) + \operatorname{sp} P_k V\right] = \frac{1}{12\pi} \iint\limits_{\Pi} V^2(x,y)dxdy.$$

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References

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