Approximation properties of some summation methods in the Smirnov classes with variable exponent

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Abstract: Let $G \subset \mathbb{C}$ be a finite domain in the complex plane, bounded by a rectifiable Jordan curve Γ. The variable exponent Lebesgue spaces $L^{p(\cdot)}(\Gamma)$ for a given Lebesgue measurable variable exponent $p(z) \geq 1$ on Γ we define as the set of Lebesgue measurable functions f, such that $\int_\Gamma |f(z)|^{p(z)} |dz| < \infty$. The function

$$\|f\|_{L^{p(\cdot)}(\Gamma)} := \inf \left\{ \lambda > 0 :_\Gamma |f(z)/\lambda|^{p(z)} |dz| \leq 1 \right\}$$

defines a norm on $L^{p(\cdot)}(\Gamma)$.

Given Lebesgue measurable function $p(\cdot) : \Gamma \to [1, \infty)$ we define the variable exponent Smirnov classes of analytic functions in G as $E^{p(\cdot)}(G) := \left\{ f \in E^1(G) : f \in L^{p(\cdot)}(\Gamma) \right\}$.

Each function $f \in E^{p(\cdot)}(G)$, has the non-tangential limits almost everywhere $(a.e)$ on Γ and hence if we define $\|f\|_{E^{p(\cdot)}(G} := \|f\|_{L^{p(\cdot)}(\Gamma)}$, then the space $E^{p(\cdot)}(G)$ is also a normed space of analytic functions in G.

In this work we continue our investigations [1,2], on the approximation problems in the variable exponent Smirnov classes $E^{p(\cdot)}(G)$. Namely, we study the approximation properties of the different approximation aggregates and obtain the appropriate estimations in term of the higher modulus of smoothness for a given function $f \in E^{p(\cdot)}(G)$.

Keywords: Variable exponent Smirnov classes, Faber series, De Vallée Poussin means, Jackson means

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