Hyperimmunity and A-computable universal numberings

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Abstract: Whether there exists a computable universal numbering for a computable family is the key question in theory of numberings. In a very general setting, this problem was explored in [1]. For sets A that are Turing jumps of the empty set, the problem was treated in [2] and other papers.

It is well known that, in the classical case, every finite family of c.e. sets has a universal (principal) computable numbering.

S.A. Badaev and S.S. Goncharov have showed that for every set A such that $\emptyset' \leq_T A$, a finite family S of A-c.e. sets has an A-computable universal numbering if and only if S contains the least set under inclusion. But also they have showed that for an infinite family of A-c.e. sets if $\emptyset' \leq_T A$ then the presence of the least set under inclusion is neither necessary nor sufficient to have an A-computable universal numbering, [3].

We considered a family F of total functions, computable relative to oracle A. Taking into the account an infinite set A is hyperimmune iff no recursive function majorizes A, [4], we have showed that if A is a hyperimmune set and if A-computable (finite or infinite) family F of total functions contains at least two elements, then F has no universal A-computable numbering. However, if Turing degree of a set A is hyperimmune-free then every A-computable finite family of total functions has A-computable universal numbering.

Keywords: universal numbering, hyperimmune set, hyperimmune-free degree

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