On boundary value problem for the fourth order mixed type equation with fractional derivative

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In the present work we investigate nonlocal problem for the fourth order mixed type equations.

Let $\Omega = \{(x,t) : 0 < x < 1, -p < t < q\}, \Omega^+ = \Omega \cap (t > 0), \Omega^- = \Omega \cap (t < 0)$, where $p, q > 0$. In the domain Ω we consider equation

$$Lu(x,t) = f(x,t) \tag{1}$$

where

$$Lu(x,t) = \begin{cases} \frac{\partial^4 u}{\partial x^4} +_C D_{0t}^{\alpha} u & t > 0, \\ \frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2}, & t < 0, \end{cases}$$

${}_C D_{0t}^{\alpha} u$ is the Caputo fractional operator of the order $\alpha \in (0,1]$

Problem A. Find a function $u(x,t)$, which is:

1) continuous in $\overline{\Omega}$, together with derivatives appearing in boundary conditions;

2) satisfies equation (1) in domains $\Omega^+ \cup \Omega^-$;

3) satisfies boundary conditions

$$u(0,t) = u_{xx}(1,t) = 0, u_x(1,t) = u_x(0,t), u_{xxx}(1,t) = u_{xxx}(0,t), -p \le t \le q, \tag{2}$$

$$u(x,-p) = 0, 0 \le x \le 1; \tag{3}$$

4) satisfies gluing condition

$${}_C D_{0t}^{\alpha} u(x,+0) = \frac{\partial u(x,-0)}{\partial t}, 0 < x < 1. \tag{4}$$

The main result of this study is the following theorem.

Theorem. Let $f(x,t) \in C_{x,t}^{4,0}(\bar{\Omega})$, $\frac{\partial^5 f(x,t)}{\partial x^5} \in L_2(\Omega)$,

$f(0,t) = f_{xx}(1,t) = f_{xxx}(0,t) = 0, f_x(1,t) = f_x(0,t), f_{xxx}(1,t) = f_{xxx}(0,t)$, and $\Delta_n = \lambda_n^2 \sin \lambda_n^2 p + \cos \lambda_n^2 p \neq 0$ at $n = 1, 2, ...$

Then the solution of Problem A exists and unique.