On a problem of the Frankl type for an equation of the mixed parabolic-hyperbolic type

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Abstract: In the paper a new non-local boundary value problem for an equation of the parabolic-hyperbolic type is formulated. This equation is of the first kind, that is, the line of type change is not a characteristic of the equation. The suggested new non-local condition binds points on boundaries of the parabolic and hyperbolic parts of the domain with each other. This problem is generalization of the well-known problems of Frankl type. Unlike the existing publications of the other authors related to the theme, in the suggested formulation of the problem the hyperbolic part of the domain coincides with a characteristic triangle. Unique solvability of the formulated problem is proved in the sense of classical and strong solutions.

Let $\Omega \subset R^2$ be a finite domain bounded for $y > 0$ by the segments AA_0, A_0B_0, B_0B, $A = (0, 1)$, $B_0 = (1, 1)$, $B = (1, 0)$ and for $y < 0$ by the characteristics $AC: \ x + y = 0$ and $BC: \ x - y = 1$ of an equation of the mixed parabolic-hyperbolic type

(1) $$Lu = \left\{ \begin{array}{c} u_x - u_{yy}, \ y > 0 \\ \\ u_{xx} - u_{yy}, \ y < 0 \end{array} \right\} = f(x, y).$$

Problem F. *Find a solution to Eq.*(1) *satisfying classical boundary conditions*

(2) $$u|_{AA_0} = 0, \quad u_y|_{A_0B_0} = 0,$$

and a non-local boundary condition

(3) $$\alpha u\left(\theta_0\left(t\right)\right) + \beta u\left(\theta_1\left(t\right)\right) = \gamma u\left(\theta\left(t\right)\right), \quad 0 \leq t \leq 1,$$

where $\theta\left(t\right) = (t, 1)$, $\theta_0\left(t\right) = \left(\frac{t}{2}, -\frac{t}{2}\right)$, $\theta_1\left(t\right) = \left(\frac{t+1}{2}, \frac{t-1}{2}\right)$; α, β *and* γ *are given numbers.*

Keywords: non-local boundary conditions, equation of the parabolic-hyperbolic type, Green's function, Frankl problem

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