On a heat transfer model for the locally inhomogeneous initial data

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Abstract: We consider a model case of the problem of heat diffusion in a homogeneous body with a special initial state. The peculiarity of this initial state is its local inhomogeneity. That is, there is a closed domain Ω inside a body, the initial state is constant out of the domain. Mathematical modeling leads to the problem for a homogeneous multi-dimensional diffusion equation.

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with sufficiently smooth boundary $\partial\Omega$. Through D we denote a cylindrical domain $D = \Omega \times (0, T)$. In D we consider a surface heat potential

$$u = \int_{\Omega} \varepsilon_n(x - \xi, t) u_0(\xi) d\xi, \tag{1}$$

where $\varepsilon_n(x,t) = \theta(t)(2a\sqrt{\pi t})^{-n} e^{-\frac{|x|^2}{4a^2 t}}$ is a fundamental solution of the heat equation

$$\Diamond u \equiv \left(\frac{\partial}{\partial t} - a^2 \triangle_x\right) u = 0. \tag{2}$$

We construct the boundary conditions on the boundary of the domain Ω, which can be characterized as "transparent" boundary conditions . We separately consider a special case – a model of redistribution of heat in a uniform linear rod, the side surface of which is insulated in the absence of (internal and external) sources of heat and of locally inhomogeneous initial state.

Throughout this note we mainly use techniques from our works [1, 2].

Keywords: diffusion equation, homogeneous body, initial state, local inhomogeneity, transparent boundary conditions

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References

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