Weighted multiplicative estimate for norm of discrete Hardy operator

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Abstract: Let $f \geq 0$ be a sequence of real numbers $f = \{f_i\}_{i=1}^{\infty}$ with non-negative terms.

Let $v > 0$, $u \geq 0$, and $w \geq 0$ be weight sequences. Let A be a matrix operator in the form:

$$(Af)_i = \sum_{j=1}^{i} a_{i,j} f_j, \ i \geq 1,$$

where $a_{i,j} \geq 0$ for $i \geq j \geq 1$ and $a_{i,j} = 0$ for $i < j$.

We consider the following weighted multiplicative inequality:

$$\|uPf\|_q \leq C\|vf\|_p^{\alpha} \ \|wAf\|_r^{1-\alpha}, \ f \geq 0, \ 0 \leq \alpha \leq 1, \tag{1}$$

where $(Pf)_i = \sum\limits_{j=1}^{i} f_j$ is the discrete Hardy operator and $\| \cdot \|_q$ is the standard norm of the space l_q.

When $\alpha = 1$ inequality (1) is a weighted estimate of the discrete Hardy operator, which is well-studied for all values of the parameters $0 < p, q < \infty$ and has numerous applications (see [1, 2]). Multiplicative inequalities are also of great importance and have applications to different problems of Analysis. Some of applications can be found in [3].

When $\alpha = 0$ from (1) we get the inequality that is of independent interest.

Keywords: multiplicative inequality, Hardy-type inequality, matrix operator, sequence

2010 Mathematics Subject Classification: 26D10, 26D15, 39B82

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