Weighted additive estimate for norm of discrete Hardy operator

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Abstract: Let $f \geq 0$ be a sequence of real numbers $f = \{f_i\}_{i=1}^{\infty}$ with non-negative terms.

Let $v > 0$, $u \geq 0$, and $w \geq 0$ be weight sequences. Let P be a discrete Hardy operator $(Pf)_i = \sum\limits_{j=1}^{i} f_j$, and A be a matrix operator in the form $(Af)_i = \sum\limits_{j=1}^{i} a_{i,j} f_j$, $i \geq 1$, where $a_{i,j} \geq 0$ for $i \geq j \geq 1$ and $a_{i,j} = 0$ for $i < j$.

We consider the following weighted additive estimate:

$$\|uPf\|_q \leq C\left(\|vf\|_p + \|wAf\|_r\right), \ f \geq 0, \tag{1}$$

where $\|\cdot\|_q$ is the standard norm of the space l_q.

In papers [1–3] under some conditions on the elements $(a_{i,j})$ the authors have found necessary and sufficient conditions for the validity of the inequality:

$$\|uAf\|_q \leq C\left(\|vf\|_p + \|wPf\|_p\right), \ f \geq 0,$$

where $1 < p, q < \infty$.

Moreover, continuous analogue of inequality (1) has been studied in paper [3] for $A \equiv P$, $r = p$ and $1 < p \leq q < \infty$.

Keywords: additive inequality, Hardy-type inequality, matrix operator, sequence

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