On first eigenvalue of Laplace operator

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Abstract: We consider a self-adjoint differential operator in a Hilbert space. Then the domain of the operator is changed by the perturbation of the boundary conditions so that a given neighborhood "there is no eigenvalues on neighborhood of zero" from the points of the spectrum of the perturbed operator. For the Sturm-Liouville operator on the segment and the Laplace operator on the square such a possibility is achieved through integral perturbations of boundary conditions. These statements are given with full proves and with possible extension. The G.G. Islamov's paper [1] solves the following problem. Suppose that the closed operator A of the Banach space X has eigenvalues in a "prohibited" area of Ω the complex plane. The requirement is for a K finite-dimensional $V = A - K$ operator, wherein the operator will not have points in the spectrum in Ω area. Our main result is following theorem:

Theorem 1. If the parameter α chosen so, that inequality holds

$$(\lambda_2 - \lambda_1) \leq \alpha \int_0^\pi \frac{\partial v_1(x_1, \pi)}{\partial x_2} dx_1, \tag{1}$$

the eigenvalues $\{\eta_k\}_{k=1}^{\infty}$ of the operator B_1 defined by the formula

$$\eta_k = \lambda_k \text{ when } k \geq 2,$$

and η_1 is the only real root of the equation.

Keywords: Laplace operator, eigenfunction, eigenvalue.

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References

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