# An isoperimetric inequality for heat potential and heat equation

Makhmud A. SADYBEKOV [1], Aidyn A. KASSYMOV [2]

*E-mail: sadybekov@math.kz*[1]*, kassymov@math.kz*[2]
*Institute of Mathematics and Mathematical Modeling, Kazakhstan*

**Abstract:** In this paper we prove that the ball is a minimizer of the first characteristic number of heat potential and heat operator among all Euclidean domains of a given measure.

Historically, for the first time of scientific literature, in Rayleigh's famous book "The Theory of Sound" (first published in 1877), by using some explicit computation and physical interpretations, he stated that a disk minimizes (among all domains of same area) the first eigenvalue of the Dirichlet Laplacian and the norm of an inverse operator is maximized in a disk among all Euclidean domains of a given area. The proof of this conjecture was obtained after about 50 years later, simultaneously (and independently) by G. Faber and E. Krahn. Nowadays, the Rayleigh-Faber-Krahn inequality has been extended to many other operators. In this paper we prove the Rayleigh-Faber-Krahn theorem for the heat potential operator $H$, i.e. it is proved that the ball is a minimizer of the first characteristic number of the integral operator $H$ among all domains of a given measure in $R^d$. The main reason why the result is useful, beyond the intrinsic interest of geometric extremum problems, is that it gives a priori bound for operator norm of the heat potential on arbitrary cylindric domains.

**Keywords:** heat potential, heat equation, Rayleigh-Faber-Krahn inequalty, Schatten p-norm, isoperimetric inequality.

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