Solvability of a stationary problem of magnetohydrodynamics

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Abstract: This work is devoted to study the following stationary problem of magnetohydrodynamics consisting in finding the functions $\vec{v}(x)$, $p(x)$, $\vec{H}(x)$ and $\vec{E}(x)$ in $\Omega \subset R^3$:

$$(1)\quad -\nu\Delta\vec{v}+(\vec{v}\cdot\nabla)\,\vec{v}-\frac{\mu}{\rho}\left(\vec{H}\cdot\nabla\right)\vec{H}+\frac{1}{\rho}\nabla\left(p\,(x)+\frac{\mu}{2}\left|\vec{H}\right|^2\right)=\vec{f}\,(x)\,,\ \ x\in\Omega,$$

$$(2)\qquad div\vec{v}\,(x)=0,\quad x\in\Omega,$$

$$(3)\qquad rot\vec{H}\,(x)-\sigma\left(\vec{E}\,(x)+\mu\left[\vec{v}\times\vec{H}\right]\right)=\vec{j}\,(x)\,,\quad x\in\Omega,$$

$$(4)\qquad div\mu\vec{H}\,(x)=0,\quad x\in\Omega,$$

$$(5)\qquad rot\vec{E}\,(x)=0,\quad x\in\Omega,$$

$$(6)\qquad \vec{v}\,(x)\,|_S=0\,,\vec{E}_\tau\,(x)\,|_S=0\,,\vec{H}\cdot\vec{n}\,|_S=0\,.$$

Here $\vec{n}$ is the unit outward normal to S, and $\vec{E}_\tau=\vec{E}-\vec{n}\left(\vec{n}\cdot\vec{E}\right)$. $\Omega\subset R^3$ is the bounded domain with smooth boundary S.

Using the results in [1]- [3], we prove unique solvability of (1)-(6) in Sobolev and Hölder spaces.

Keywords: magnetohydrodynamics, generalized solution, stationary problem

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