On solvability of boundary value problem of magnetic gas dynamics with cylindric and spherical symmetries

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Abstract: In this paper we investigate the initial-boundary value problems for the equations motion of a viscous heat-conducting gas, taking into account the magnetic field. The main attention is paid to the case of solutions with cylindrical and spherical symmetry. The paper describes in detail the transition from the basic three-dimensional equations to cylindrical and spherical coordinate system and prove a number of theorems on the existence and uniqueness of the initial-boundary value problems.

$$\frac{\partial \rho}{\partial t} + \rho^2 \frac{\partial (x^m u)}{\partial q} = 0, \ P = R\rho\theta,$$

$$\frac{\partial u}{\partial t} + \frac{H^2}{x\rho} = x^m \frac{\partial}{\partial q}\left(\rho \frac{\partial (x^m u)}{\partial q}\right) - x^m \frac{\partial P}{\partial q} - x^m H \frac{\partial H}{\partial q},$$

$$\frac{\partial \theta}{\partial t} = æ \frac{\partial}{\partial q}\left(x^{2m}\rho \frac{\partial \theta}{\partial q}\right) + \rho \left[\frac{\partial (x^m u)}{\partial q}\right]^2 +$$

$$+\rho x^{2(m-1)} \left[\frac{\partial (xH)}{\partial q}\right]^2 - P\frac{\partial (x^m u)}{\partial q} - \frac{3m}{2}\frac{\partial (x^{m-1}u^2)}{\partial q}, \tag{1}$$

$$\frac{\partial H}{\partial t} + x\rho H \frac{\partial (x^{m-1}u)}{\partial q} = x\rho \frac{\partial}{\partial q}\left(\rho x^{2(m-1)} \frac{\partial (xH)}{\partial q}\right),$$

$$(u, \rho, \theta, H)(q, 0) = (u_0, \rho_0, \theta_0, H_0)(q), q \in [0, b], \tag{2}$$

$$(u, \frac{\partial \theta}{\partial q}, H)(0, t) = (u, \frac{\partial \theta}{\partial q}, H)(b, t) = 0, t \in [0, T]. \tag{3}$$

This article explores the initial-boundary value problems (1)-(3) for the equations motion of a viscous heat-conducting gas, taking into account the magnetic field with cylindrical and spherical symmetry. The paper describes in detail the transition from the basic three-dimensional equation in cylindrical and spherical coordinates. Under certain conditions the data (2) prove a number of theorems on the existence and uniqueness of the initial-boundary value problems.

Keywords: gas dynamics equation, boundary value problems, spherical symmetry

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