On Spectral Properties of Perturbations of the Operator $-u''(-x)$ with Initial Data

Leonid KRITSKOV [1], Abdizhahan SARSENBI [2,3]

[1] *Moscow State University named after M.V.Lomonosov, Moscow, Russia*
E-mail: kritskov@cs.msu.su
[2] *South-Kazakhstan State University, Shymkent, Kazakhstan*
[3] *Institute for Mathematics and Mathematical Modeling, Almaty, Kazakhstan*
E-mail: abzhahan@mail.ru

Abstract: In the recent years there appears a rising interest to the problems that contain involution in the main terms of its differential operations [1–3].

The peculiarity of the initial value problem

$$L_0u = -u''(-x),\ -1 < x < 1, \quad u(-1) = u'(-1) = 0,$$

for the simplest second-order differential operation with involution (reflection) is the non-emptiness of its spectrum. As the related spectral expansion could be constructed explicitly, it is interesting to study the spectral expansion for the perturbed operator $L = L_0 + Q$ where $Qu = q(x)u(x)$ is the multiplication on a square integrable complex-valued potential $q(x)$.

Let $S_m(x, f)$ and $\sigma_m(x, f)$ be m-th partial sums of the spectral expansions for L_0 and L respectively. Using asymptotics of the Green's function for L, the equiconvergence property is obtained, i.e.

$$S_m(x, f) - \sigma_m(x, f) = o(1), \quad m \to \infty,$$

uniformly for $x \in [-1, 1]$ and any $f(x) \in L_1(-1, 1)$. It directly provides the claim that $\sigma_m(x, f)$ converges to $f(x)$ in the L_2-metric for any $f(x) \in L_2(-1, 1)$, or, in other words, the root functions of L constitute basis.

The research is supported by the grant 0971/GF4 and by the target program 0085/PTSF-14 from the Ministry for Education and Science of the Republic of Kazakhstan.

Keywords: ODE with involution, spectral expansion, equiconvergence

2010 Mathematics Subject Classification: 34K08, 34L10

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