Differentiation of the functional in an optimization problem for diffusion and convective transfer coefficients of elliptic imperfect contact interface problems

Aigul MANAPOVA [1][1]

[1] *Department of Mathematics and IT, Bashkir State University, Ufa, Republic of Bashkortostan, Russia*
E-mail: aygulrm@mail.ru

Abstract:

Convection-diffusion problems are typical for mathematical models of liquid and gas mechanics, since heat and impurities transfer can occur not only due to diffusion, but also to the motion of the medium (see [1]). Currently, the most profound results in the theory of numerical solution of problem for PDEs and optimal control problems are obtained for processes with self-adjoint operators.

In this work we investigate issues of numerical solving of optimal control problems for second order elliptic equations with non-self-adjoint operators - convection-diffusion problems. Control processes are described by semi-linear convection-diffusion equation with discontinuous data and solutions (states) subject to the boundary interface conditions of imperfect type (i.e., problems with a jump of the coefficients and the solution on the interface; the jump of the solution is proportional to the normal component of the flux). Controls are involved in the coefficients of diffusion and convective transfer.

The subject of this paper is related to [2]. We prove differentiability and Lipshitz continuity of the cost functional, depending on a state of the system and a control. The calculation of the gradients uses the numerical solutions of direct problems for the state and adjoint problems.

Keywords: optimal control, semi-linear elliptic equation, imperfect contact, convective transfer, cost functional, differentiability

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