On a homogeneous parabolic problem in an infinite corner domain

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Abstract: It is considered the homogeneous boundary problem

$$\frac{\partial u(x,t)}{\partial t} - a^2 \frac{\partial^2 u(x,t)}{\partial x^2} = 0, \ \{x,t\} \in G = \{x,t:\ 0 < x < t,\ t > 0\}; \tag{1}$$

$$\frac{\partial u(x,t)}{\partial x}|_{x=0} = 0, \ \frac{\partial u(x,t)}{\partial x}|_{x=t} + \frac{d\tilde{u(t)}}{dt} = 0; \tag{2}$$

where $\tilde{u}(t) = u(t,t)$.

Note that problem (1)–(2) is homogeneous case of the problem, studied in [1], which stated that the case of non-homogeneous boundary value problem "... it appears to be useful in the study of some free boundary value problems. "In the paper [1] was obtain a theorem on the unique solvability of the non-homogeneous boundary value problem in weighted Holder spaces.

In this paper, we set in class of essentially bounded functions with a given weight the existence of a nontrivial solution for a constant factor and a constant term. We introduce the class as follows:

$$(x + t^{1/2})^{-1} u(x,t) \in L_\infty(G), \text{ i.e. } u(x,t) \in L_\infty(G;\ (x + t^{1/2})^{-1}).$$

Theorem 1. *The boundary value problem (1)–(2) has a nontrivial solution* $u(x,t) = C_2\, \tilde{u}(x,t) + C_1$, *where* $\tilde{u}(x,t) \in L_\infty(G;\ (x + \sqrt{t})^{-1}),\ \ C_1,\, C_2 = const.$

Theorem 2. *In the class of functions* $L_\infty\left(G;\ [x^{1+\alpha} + t^{(1+\alpha)/2}]^{-1}\right)$ *the boundary value problem (1)–(2) has only the trivial solution* $u(x,t) \equiv 0$.

Keywords: parabolic equation, boundary value problem, integral equation

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References

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