Convergence of kernel density estimators on compact sets

Kairat Mynbaev [1,2,3]

[1] *International School of Economics, Kazakh-British Technical University, Almaty, Kazakhstan*
E-mail: kairat_mynbayev@yahoo.com
[2] *Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan*
[3] *Kazakh National University, Almaty, Kazakhstan*

Abstract: In many statistical applications, one has to estimate the rate of convergence in probability of a kernel density estimator. The usual estimate on compact sets is of order $\left(\frac{\log n}{nh}\right)^{1/2}$ where n is the sample size, $h > 0$ is a bandwidth that depends on n and $h \to 0$, $nh \to \infty$ [1]. We improve this estimate by replacing $\log n$ with a function of n which grows slower than $\log n$. For example, one can take the kth iterated log, for any natural k. The idea is illustrated in estimating a density in the one-dimensional case and can be used in many other setups.

Let K be a kernel, that is a function on R satisfying $\int_R K(t)dt = 1$. By $X_1, ..., X_n$ we denote independent observations from a density f. A kernel density estimate of $f(x)$ is defined by $\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^n K\left(\frac{x-X_i}{h}\right)$.

Theorem 0.1. *Let the kernel K satisfy the Lipschitz condition on R, be bounded and let $\int_R K^2(t)(1+|t|)dt < \infty$. Assume that the density f also satisfies the Lipschitz condition on R and is bounded. Fix a compact $G \subset R$ and let f be positive on G. Suppose a function $\psi(n)$ is such that $\psi(n) \to \infty$ as $n \to \infty$ and $\limsup_{n\to\infty} \frac{n^2}{\psi(n)} < \infty$, $\liminf \frac{nh}{\psi(n)} > 0$. Then*

$$\sup_{x\in G}\left|\hat{f}(x) - E\hat{f}(x)\right| = O_p\left(\left(\frac{\psi(n)}{nh}\right)^{1/2}\right).$$

Keywords: kernel density estimator, convergence in probability.

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References

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