A conditional stability estimate of continuation problem for the Helmholtz equation

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Abstract: In this paper we consider the continuation problem for the Helmholtz equation. The main result is a conditional stability estimate for a solution to the considered problem. The estimate shows that the closer solution to the surface is more stable.

We consider the initial boundary value problem for the Helmholtz equation in the domain $\Omega = (0, l) \times (0, \pi)$ [1]:

$$u_{xx} + u_{yy} + k^2 u = 0, \qquad (x, y) \in \Omega, \tag{1}$$

$$u_x(0, y) = 0, \quad u(0, y) = f(y), \qquad y \in [0, \pi], \tag{2}$$

$$u_y(x, 0) = u_y(x, \pi) = 0, \qquad x \in [0, l], \tag{3}$$

where k, l are given constants. It is required to find a function $u(x, y)$ in Ω from $f(y)$. The main theoretical result is the following

Theorem. Assume that for $f \in L_2(0, \pi)$ there exists a solution $u \in L_2(\Omega)$ to the problem (1) – (3), then the following estimate holds [2]:

$$\|u\|^2(x) \le \left(\|q\|^2 + \left|\|f_y\|^2 - \frac{k^2}{2}\|f\|^2\right|\right)^{\frac{x}{l}} \left(\|f\|^2 + \left|\|f_y\|^2 - \frac{k^2}{2}\|f\|^2\right|\right)^{\frac{l-x}{l}} e^{2x(l-x)} - \left|\|f_y\|^2 - \frac{k^2}{2}\|f\|^2\right|,$$

where $\|u\|^2(x) = \int_0^\pi u^2(x, y)dy$.

Keywords: Helmholtz equation, stability estimate, inverse problem

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References

[1] Kabanikhin, S. (2011). Inverse and Ill-posed Problems. Theory and Applications. Berlin, Boston: De Gruyter. 2011

[2] Lavrent'ev, M. M., Savel'ev, L. Y. *Operator Theory and Ill-Posed Problems*, De Gruyter, Germany, 2011, 680 pp.