Integral operators with two variable integration limits on the cone of monotone functions

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Abstract: The work is done together with A.A. Kalybay.

Let $I = (a, b)$, $-\infty \leq a < b \leq \infty$, and $1 < p, q < \infty$. Let ω and v be non-negative and measurable functions almost everywhere finite on I such that $\omega^{-q'}$, ω^{q}, $v^{p'}$ and $v^{-p'}$ are locally summable on I.

Let $M\downarrow$ and $M\uparrow$ be sets of functions non-increasing and non-decreasing on I, respectively.

For the integral operators

$$K_{-}f(x) = \int\limits_{\alpha(x)}^{\beta(x)} K(s,x)f(s)ds \text{ and } K_{+}f(x) = \int\limits_{\alpha(x)}^{\beta(x)} K(x,s)f(s)ds$$

we consider the inequalities

$$\|\omega K_{-}f\|_q \leq C\|vf\|_p, \ f \in M\downarrow, \tag{1}$$

$$\|\omega K_{+}f\|_q \leq C\|vf\|_p, \ f \in M\uparrow, \tag{2}$$

where $\|\cdot\|_p$ is the standard norm of the space $L_p(I)$. Moreover, the boundary functions α and β satisfy the following conditions:

(i) $\alpha(x)$ and $\beta(x)$ are functions differentiable and strictly increasing on I;

(ii) $\alpha(x) < \beta(x)$ for any $x \in I$; $\lim\limits_{x\to a^+} \alpha(x) = \lim\limits_{x\to a^+} \beta(x) = a$ and $\lim\limits_{x\to b^-} \alpha(x) = \lim\limits_{x\to b^-} \beta(x) = b$.

In the case when $K(s, x) \equiv 1$ the operator $K_{-}f(x)$ is called the Hardy-Steklov operator. Weighted inequalities of the form (1) and (2) for the Hardy-Steklov operators have been investigated with success [1]. The similar problem for operators with kernels has remained unsolved up to now. The aim of this work is to get necessary and sufficient conditions for the validity of inequalities (1) and (2) when the kernels $K(\cdot, \cdot)$ are from a large class.

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References

[1] Stepanov, V.D., Ushakova, E.P., "Hardy operator with variable limits on monotone functions", *J. Funct. Spaces Appl.*, Vol. 1, no. 1, pp.1–15, 2003.