Additive and multiplicative weighted estimates of intermediate integral operator

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Abstract: The work is done together with A.A. Kalybay.

We consider the inequalities

$$\|uK_\beta f\|_q \le C\left(\|\rho f\|_p + \|vKf\|_r\right), \quad f \ge 0, \tag{1}$$

$$\|uK_\beta f\|_q \le C\|\rho f\|_p^\alpha \|vKf\|_r^{1-\alpha}, \quad f \ge 0, \tag{2}$$

where $0 \le \alpha, \beta \le 1$ and $\|\cdot\|_p$ is the standard norm of the space $L_p(0,\infty)$. Moreover, K and K_β are integral operators of the form

$$Kf(x) = \int_0^\infty K(x,s)f(s)ds, \quad K_\beta f(x) = \int_0^\infty K^\beta(x,s)f(s)ds$$

with the kernel $K(\cdot,\cdot) \ge 0$ satisfying the condition

$$\exists D \ge 1 : D^{-1}\Big(K(x,t) + K(t,s)\Big) \le K(x,s) \le D\Big(K(x,t) + K(t,s)\Big)$$

for $x \ge t \ge s > 0$.

We give necessary and sufficient conditions for the validity of inequalities (**??**) and (**??**) for $1 \le \max\{p,r\} \le q \le \infty$ and $\frac{\alpha}{p} + \frac{1-\alpha}{r} \ge \frac{1}{q}$, respectively.

As an application, we obtain criteria for the inequalities

$$\|uf^{(k)}\|_q \le C\left(\|\rho f^{(n)}\|_p + \|vf\|_r\right)$$

and

$$\|uf^{(k)}\|_q \le C\|\rho f^{(n)}\|_p^\alpha \|vf\|_r^{1-\alpha}$$

to hold on the class of n-convex functions absolutely monotone on the interval $(0,\infty)$; in particular, when $(n=2)$ on the class of convex and twice differentiable functions, where $0 \le k < n$.

Keywords: multiplicative inequality, additive inequality, integral operator, kernel

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