Nonlocal estimates for solutions of a singular higher order differential equation

Kordan OSPANOV [1,2]

[1] *Faculty of Mathematics and Mechanics, L.N. Gumilyov Eurasian National University, Ástana, Kazakhstan*

[2] *Institute of Mathematics and Mathematical Modelling, Álmaty, Kazakhstan*

E-mail: ospanov_kn@enu.kz

Abstract: We consider the linear differential equation of even order

$$L_0 y := y^{(2n)} + a_1 y^{(2n-1)} + a_2 y^{(2n-2)} + ... + a_{2n} y = f(x), \tag{1}$$

where $x \in \mathbf{R} = (-\infty, +\infty)$, and $a_k = a_k(x)$ $(k = 1, 2, ..., 2n)$ and $f \in L_2 = L_2(\mathbf{R})$ are given functions.

In this paper we investigate the equation (1) and give some sufficient conditions for its unique and coercive solvability. Similar questions were studied in many papers (see for instance the monographs [1], [2] and [3]). However, their results were obtained under the following conditions: the intermediate coefficients $a_s(\, s = 1, 2, \dots, 2n-1)$ of equation (1) are constant, or their growths are bounded above by some degrees of the function $|a_{2n}|$ at infinity. In the case $n = 1$, the equation (1) has been investigated in [4].

We give an estimate of norms of solution and its derivatives up to the order $2n$ of the equation (1), in the case a_s $(s = 1, 2, \dots, 2n-1)$ are not bounded and a_{2n} can be equal to zero.

Keywords: even order differential equation, coercive solvability, intermediate coefficient

2010 Mathematics Subject Classification: 34B40

References

[1] Coddington, E.A., Levinson, N., "Theory of Ordinary Differential Equations", -*New York: McGraw-Hill*, 1955.

[2] Naimark, M.A., "Linear Differential Operators, Vols. I, II", - *New York: Ungar*, 1968.

[3] Fedoryuk, M.V., "Asymptotic analysis: linear ordinary differential equations", *Berlin: Springer*, 1993.

[4] Ospanov, K., Akhmetkaliyeva, R.D., "Separation and the existence theorem for second order nonlinear differential equation", *Electronic Journal of Qualitative Theory of Differential Equations*, No.66, pp. 1-12, 2012.